Adexa Optimization Technology

Adexa, Inc.

Optimization is a fascinating topic in mathematics and everyday life. Before I expand on this, let me take you back to when I was growing up and I was introduced to the Theorem of Donkey in elementary school. It is about the fact that the shortest distance between two points is a straight line. And even a donkey would know that and it would walk from where it is standing to its food in straight line! Obviously this is optimization of time and effort to get to what you want in shortest possible time and distance! Here the assumption is your objective is well-defined. How about, if we had a much larger and more appetizing food for the donkey at twice the distance behind the animal? And the objective is to find the highest amount and most appetizing food. This means, we have to perform a search and eliminate the sub-optimal solutions. Having found the second one, how do we know that there are no other. Do we need to continue searching or do we stop. If we decide on the latter then we might get sub-optimal solution. However if we keep going we might never find a solution! Problems of this nature are classified as intractable or semi-intractable! And there are no known mathematical solutions that can solve such problems in a polynomial manner. In other words as the size of the problem gest bigger than the time it takes to search goes up in a polynomial manner rather than exponential way.

Optimization problems in planning supply chains fall in this category of being intractable. What makes it very intriguing is that, in most cases the objective functions are not well defined. For example, if the objective is to make most profit out of sale and production of certain products, then when a new product is introduced, it might be sold at a loss to begin with in order to capture the market share. Hence the latter, market share, becomes part of the objective function. There are also many constraints that need to be considered. Some are objective such as capacity and supply issues and some are not so. As an example of the latter consider giving preferences to one product manager over another for allocation of resources and capacity or going for extra cost of overnight shipment to ensure on-time delivery to a customer!

In order to be able to respect all the millions of variables and constraints and come up with an optimal solution one needs to go beyond just mathematical optimization techniques. It becomes a separate domain of expertise to be able to design solutions that can meet the mathematical rigor as well as subjective human needs and necessary inputs to guide the search such that the solution is “truly optimal.” That means a solution that is mathematically close to optimal and at the same time meets the factors which may not be mathematically expressible. At Adexa, we have been doing this for over 2 decades. Our founders were amongst the first to develop and patent optimization techniques for complex planning and scheduling problems as they relate to manufacturing companies and beyond. Hence, *First in Optimization* is our heritage.

Techniques that we have adapted are many and they are simply ingredients of our overall solution. How we mix them and apply them combined with how we interact with the system users to enable an optimal solution is our true expertise. Some of these techniques are described below.

Enabling technologies supporting this comprehensive solution include:

- Advanced Mathematical Programming (MP) based optimization methods
- Fast optimization heuristics based on Constraint Propagation (CP)
- Stochastic Queuing Theory
- Artificial Intelligence Search techniques (simulated annealing, genetic algorithms)
- Adexa's Single Data Model
- Adexa's unique data representation

Linear Programming Based Optimization

Strategic planning problems dealing with sourcing decisions, product mix determinations, and routing configurations can be formulated much the same way as network flow problems where discrete events are aggregated and represented as continuous flows through the network. These problems can then be solved using LP techniques.

Adexa's planning engines employ this technique to formulate the initial LP model. It is possible, however, that the LP solution may require hours of computing time even on the fastest machines. To avoid this, Adexa's optimization engine takes advantage of certain characteristics of the initial system to substantially reduce the size of the LP problem. The resulting LP problem is then solved using a LP solver that is widely available. This methodology results in an order of magnitude decrease in the solution time and enables the user to perform real-time what-if analysis on the entire system as opposed to batch type analysis that could take hours at a time.

Constraint Propagation Heuristics

Although LP techniques can produce excellent results at the higher levels of planning, they are usually unsuitable at the lower levels of planning. This is due to the fact that optimization at the lower levels of planning revolves heavily around the sequencing of discrete events that cannot be effectively represented as continuous flows.

Discrete event optimization problems are in general intractable, meaning that the computation time required to obtain the optimal solution grows exponentially as the problem size grows. Therefore, *exact* solutions to even small optimization problems might require hours of computation, and larger problems are practically impossible to solve *exactly*. As a result, the objective of the optimization at this level is to find a feasible solution that is as close as possible to optimal.

Many heuristics have been developed in the recent years that attempt to improve the solution quality on these types of optimization problems. Some of these heuristics such as simulated annealing, and genetic algorithms have gained much popularity because of their success in certain areas.

These algorithms attempt to search the solution space based on a trial and error approach, trying to improve the quality of the solution at each step. In general, this type of unconstrained search will lead to a substantial overhead in run time due to the fact that many of the steps in such algorithms are redundant and will fail to improve the quality of the solution. However in certain instances they can be very effective once the solution space is contained. We do this by applying CP methodology described below.

Adexa's heuristic optimization approach, which is fundamentally based on Constraint Propagation (CP), reduces the search space at every step and therefore converges to the optimal solution in much faster time than the sole use of genetic algorithms or simulated annealing.

The Eight Queen problem in chess is a good example of how effective constraint propagation can be in speeding up the solution convergence. The objective of the Eight Queen problem is to place eight queens on the chess board in a way that no two queens can strike each other. Figures 1 through 3 show the simplified 4 queen problem on a 4X4 chess board and how constraint propagation helps speed up the solution convergence.

As the first move we choose to place the first queen in the upper left square. If we were to solve this problem using an approach such as genetic algorithms, the second move could place a queen anywhere on the chessboard. This type of trial and error approach could lead to many unsuccessful trials before an acceptable move is found.

Figure 1: Place first queen in upper left square

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A constraint propagation based algorithm will first eliminate all unacceptable moves after every step. In the Four Queen problem, the first move will result in the elimination of the top row, the leftmost column and all the squares on the diagonal passing through the top left square.

Having the search space reduced, the second step of the algorithm will only try one of the remaining six squares.

Figure 2: Search space reduced after the first move.

Yellow boxes are not acceptable next moves.

After placing the second queen on the board, once again, new constraints are propagated, further reducing the search space. In this particular example, the search space is reduced to a single point after the second move, leaving one acceptable square on the board as seen below.

Figure 3: Search space reduced further to one square after the second move

Constraint propagation techniques can lead to drastic solution time reductions when applied to the solution of scheduling and sequencing problems. As it can be seen once the problem size is reduced then other search techniques are deployed for local optimum.

Single Data Model

It is a well-known fact that in a planning problem, the sum of local optimums is not equal to the global optimum. This means that optimization of distribution plans, shop floor plans; demand plans, etc., cannot be performed in isolation.

Meaningful optimization can only be performed if all aspects of the planning problem that affect enterprise objectives are considered at the same time.

Adexa Applications' Single Data Model lays ground for a fundamentally sound optimization strategy which avoids the trap of local optimums and allows the enterprise objectives to permeate into all aspects of planning down to shop floor events.

High level objectives when propagated to the lower levels will also further constrain the solution at lower levels of planning. The outcome is similar to that of constraint propagation, i.e. at the lower levels of planning the search space is reduced by the propagation of the constraints at the higher levels. Therefore, as a side benefit, the solution time for the lower level planning problems is reduced due to the visibility into the constraints at higher levels of planning.

Data Representation

There are many instances where data representing one form of production is only slightly different from another. Most planning applications are unable to take advantage of the similarities between processes, routings, operations, etc.

Adexa's object data model is design to maximize data reuse by allowing different production processes to be defined using shared objects. For example, if a process is only different from another in a few operations, "operation overrides" can be used to represent the new process without having to redefine the entire set of operations. Also, routes can be broken into segments. A new route may only be different from an existing route in one segment, therefore, only that segment of the route has to be redefined.

Data reuse can lead to a much more compact data storage that requires considerably less memory. Smaller memory requirements also lead to faster execution times. Additionally, Adexa Applications constraint propagation heuristics take advantage of route segments to speed up the search by allowing partial backtracking within a route as opposed to backtracking all the way to the start of a route.

The combination of compact and intelligent data storage and algorithmic efficiencies based on Adexa Applications' data representation can lead to significant improvements in total solution time, accuracy and optimality.